Analyzing Quadratic Functions in General Form

Wednesday, October 23, 2019 1:54 PM

$y = -2x^2 - 10x - 12$

$y = -2(x^2 + 5x + 6)$

$= -2(x+3)(x+2)$

$y = -2\left(x^2 + 5x + \frac{25}{4}\right) - 12 - (-2)\left(\frac{25}{4}\right)$

$y = -2\left(x + \frac{5}{2}\right)^2 - 12 + \frac{25}{2}$

$y = -2\left(x + \frac{5}{2}\right)^2 + \frac{1}{2}$

$\frac{-\frac{1}{2}}{-2} = \frac{-2\left(x + \frac{5}{2}\right)^2}{-2}$

$\sqrt{\frac{1}{4}} = \sqrt{\left(x + 2\frac{1}{2}\right)^2}$

$\pm\frac{1}{2} = x + 2\frac{1}{2}$

$-2\frac{1}{2} \pm \frac{1}{2} = x$

$x = -2\frac{1}{2} - \frac{1}{2} = -3$

$-2\frac{1}{2} + \frac{1}{2} = -2$

opens down.

vertex $\left(-2\frac{1}{2}, \frac{1}{2}\right)$

axis of sym $x = -2\frac{1}{2}$

y-int $(0, -12)$

x-int $(-3, 0)$ $(-2, 0)$

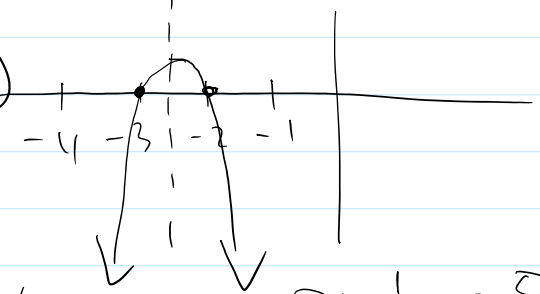
domain $x \in \mathbb{R}$

range $y \leq \frac{1}{2}$ $\left(\frac{25}{2} = 12\frac{1}{2}\right)$

vertex $\left(-2\frac{1}{2}, \frac{1}{2}\right)$ x-int $(-2, 0)$ $(-3, 0)$

x value of vertex (p)

$p = \frac{-2 + -3}{2} = -\frac{5}{2}$



y value of vertex. put $-\frac{5}{2}$ in for x

$y = -2(x+2)(x+3)$

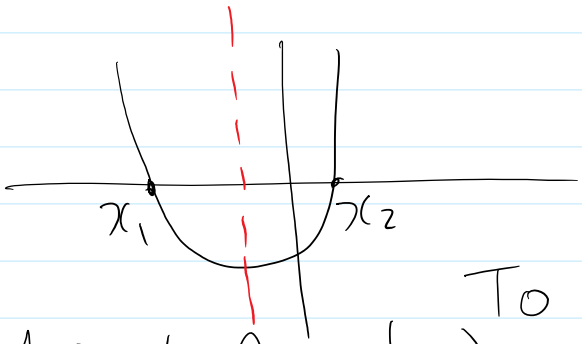
$= -2\left(-\frac{5}{2}+\frac{4}{2}\right)\left(-\frac{5}{2}+\frac{6}{2}\right)$

$= \frac{-2}{1}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{2}{4} = \frac{1}{2}$

For any quadratic function that can be factored you can find the zeroes (x-ints) and use those to find the x value of the vertex.

$y = a(x-x_1)(x-x_2)$ Factored form of equation

x_1 = 1st x-int x_2 = 2nd x-int



x value of vertex (p) = $\frac{x_1 + x_2}{2}$.

To find the y value of the vertex (q) plug in the p value into either form of equation (general or factored) for x and solve for y.

Eg Find the following for $y = \frac{1}{2}x^2 - \frac{9}{2}x + 7$

① y-int $(0, 7)$

② x-int $(7, 0)$ $(2, 0)$

③ axis of symmetry $x = 4\frac{1}{2}$

④ vertex $\left(4\frac{1}{2}, -3\frac{1}{8}\right)$

$y = \frac{1}{2}(x^2 - 9x + 14)$

$y = \frac{1}{2}(x-7)(x-2)$

$p = \frac{7+2}{2} = \frac{9}{2}$

$q = \frac{1}{2}\left(\frac{9}{2} - \frac{14}{2}\right)\left(\frac{9}{2} - \frac{4}{2}\right)$

$q = \frac{1}{2}\left(-\frac{5}{2}\right)\left(\frac{5}{2}\right) = -\frac{25}{8}$

$q = \frac{1}{2}\left(-\frac{5}{2}\right)\left(\frac{5}{2}\right) = -\frac{25}{8}$

Try Find the following for $y = -2.5x^2 - 7.5x + 10$

$y = -2.5(x^2 + 3x - 4)$

$y = -2.5(x+4)(x-1)$

① y-int $(0, 10)$

② x-int $(-4, 0)$ $(1, 0)$

③ axis of sym. $x = -1\frac{1}{2}$

④ vertex. $\left(-\frac{3}{2}, \frac{125}{8}\right)$

$\left(-1\frac{1}{2}, 15\frac{5}{8}\right)$

$p = \frac{-4+1}{2} = \frac{-3}{2}$

$q = -\frac{5}{2}\left(-\frac{3}{2} + \frac{8}{2}\right)\left(\frac{-3}{2} - \frac{2}{2}\right)$

$q = -\frac{5}{2}\left(\frac{5}{2}\right)\left(-\frac{5}{2}\right) = \frac{125}{8}$

Eg The graph of a quadratic function passes thru $(4, -5)$ and has zeroes (x-int) at -1 and 3. Write the equation of the function in general form

$y = a(x - x_1)(x - x_2)$

$y = a(x+1)(x-3)$

$-5 = a(4+1)(4-3)$

$-5 = a(5)(1)$

$\frac{-5}{5} = \frac{5a}{5}$

$-1 = a$

$y = -1(x+1)(x-3)$

$y = -1(x^2 - 2x - 3)$

$y = -x^2 + 2x + 3$

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