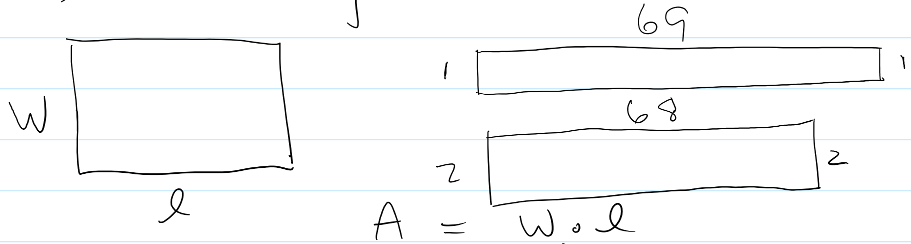
Modelling + Solving Problems with Quadratic Functions

Monday, October 28, 2019 10:04 AM

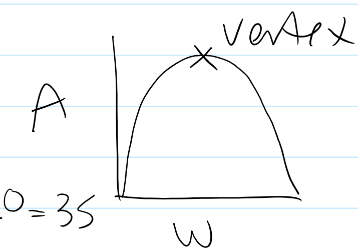
Eg A gardener wants to fence a rectangular garden to keep out the deer. She has 140m of fencing. What are the dimensions of the largest garden she can fence?



$A = 69m^2$

$A = 136m^2$

$A = w \cdot l$



$2l + 2w = 140$ $\quad A = w(70 - w)$

$\frac{2l}{2} = \frac{140 - 2w}{2}$ $\quad$ zeroes: 0, 70

$l = 70 - w$ $\quad$ w value of vertex $= \frac{0+70}{2} = 35$

$l = 70 - 35 = 35$

She'el get the biggest garden with dimensions 35m x 35m.

$A = w(70 - w)$
$0 = w(70 - w)$
$w = 0$
$70 - w = 0$
$w = 70$

Eg 2 numbers have a sum of 12. Does the sum of their squares have a maximum or minimum? Find this value & the numbers?

Let S = Sum of squares.
Let x = 1st number
$12 - x$ = 2nd number

$$S = x^2 + (12 - x)^2$$

$S = x^2 + (144 - 24x + x^2)$

$S = 2x^2 - 24x + 144$ minimum sum

$S = 2(x^2 - 12x + 36) + 144 - 2(36)$

$S = 2(x-6)^2 + 72.$

vertex $(6, 72)$

The 1st # is 6

The 2nd # is $12 - 6 = 6$

The minimum sum is 72.

Try: 2 numbers have a difference of 10. Does their product have a maximum or minimum? Find its value and the numbers.

Let x = smaller # $\quad P = x(x+10)$

$x + 10$ = larger # $\quad$ zeroes: $0 \quad -10$

P = product $\quad$ x value of vertex $\frac{0+-10}{2} = -5$

$-5 + 10 = 5$

Numbers are -5 and 5. The minimum is -25.

$P = x(x+10)$

$P = (x^2 + 10x + 25) - 25$

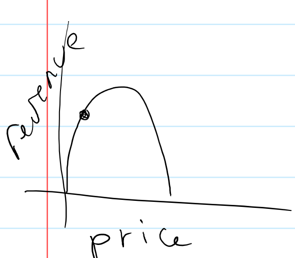
$P = (x+5)^2 - \underline{\underline{25}}$

$x = -5$

$-5 + 10 = 5.$

$P = -25$

Eg Yearbooks cost \$32. At that price 600 students purchase them. For every \$2 increase in price 25 less students will purchase books. At what price will maximum revenue be made? What is this revenue?



Revenue = price of item × # items sold

Let R = Revenue

x = # of \$2 increases in price

Revenue = price × # sold

$= 32 \times 600 \quad (19200)$

$34 \times 575 \quad (19550)$

$$R = (32 + 2x)(600 - 25x)$$

zeroes $\quad 32 + 2x = 0 \qquad 600 - 25x = 0$

$2x = -32 \qquad -25x = -600$

$x = -16 \qquad x = 24$

x value of vertex $= \dfrac{-16 + 24}{2} = \dfrac{8}{2} = 4$

New price $= 32 + 2(4) = 40$

sold $= 600 - 25(4) = 500$

Max. Revenue $= 40 \times 500 = \$20{,}000$

Pg 343-345 # 3-10.

$$y = -35(x + 100)^2 - 1200$$

④ P = product.

Let x = smaller #

$x + 6$ = larger #

$y - x = 6$

$y = \boxed{x + 6}$

$\boxed{x+6}$ = larger # $y = \boxed{x+6}$

$P = x(x+6)$

zeroes: $x = 0, -6$

x value of vertex $\frac{0+-6}{2} = -3$

larger # $= -3+6 = 3$

product $= -3 \cdot 3 = -9$

$P = x(x+6)$

$P = x^2 + 6x + 9 - 9$

$P = (x+3)^2 - 9$

$(-3, -9)$

x, P

x

$12 - x$

(6) Let x = smaller #

$x + 18$ = larger #

S = sum of squares

$S = x^2 + (x+18)^2$

$S = x^2 + x^2 + 36x + 324$

$S = 2x^2 + 36x + 324$

$S = 2(x^2 + 18x + 81) + 324 - 2(81)$

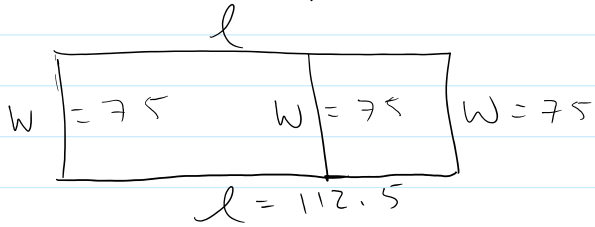
$S = 2(x+9)^2 + 162$. vertex $(-9, 162)$

1st # -9

2nd # $+9$

Sum of squares $= 162$

(9)



$3w + 2l = 450$

$\frac{2l}{2} = \frac{450 - 3w}{2}$

$l = 225 - \frac{3}{2}w$

$A = l \cdot w$

$A = (225 - \frac{3}{2}w) \cdot w$

Zeroes: $225 - \frac{3}{2}w = 0$ $\quad$ $\underline{w = 0}$ $\quad$ w value of vertex

$2\left(-\frac{3}{2}w\right) = (-225)2$ $\quad$ $W = \frac{0+150}{2} = 75$

$\frac{-3w}{-3} = \frac{-450}{-3}$ $\quad$ $l = 225 - \frac{3}{2}(75)$

$l = 225 - 112.5$

$\underline{w = 150}$ $\quad$ $l = 112.5$

Max area from dimensions of 112.5m x 75m

(10) $R = .95 \times 10,000$

R = revenue.

Let x = the # of $\$.15$ increases in price

R = price × # sold.

$R = (.95 + .15x)(10,000 - 500x)$

Zeroes: $.95 + .15x = 0$ $\quad$ $10,000 - 500x = 0$

$.15x = -.95$ $\quad$ $10,000 = 500x$

$x = -6.\overline{3}$ $\quad$ $20 = x$

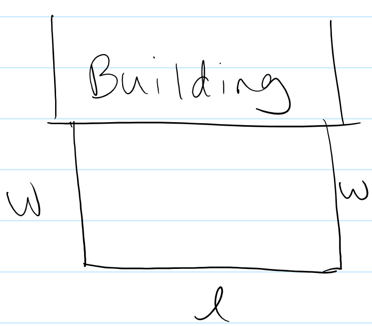
x value of vertex $= \frac{-6.\overline{3} + 20}{2} = \underline{6.8} \Rightarrow 7$ increases

Price $= .95 + .15(7)$ $\quad$ # sold $10,000 - 500(7)$

$= \$\underline{\underline{2.00}}$ $\quad$ 6500

Max Revenue $= \$2.00 \times 6500 = \$13,000$

(8)



$2w + l = 800$

$l = 800 - 2w$

$A = (800 - 2w)w$

zeroes: 400, 0

$w = \frac{400+0}{2} = 200$

$l = 800 - 2(200) = 400$