## **Pre-Calc 11 Final Review**

## Short Answer

1. Solve.

$$\frac{5}{h+2} - \frac{5h-14}{h^2 - h - 6} = \frac{h}{h-3}$$

- 2. Devon travels 120 km to Edmonton by car, and then returns by bus. The average speed of the car is 15 km/h greater than the average speed of the bus. If Devon's total travel time is 216 min, what is the average speed of the bus?
- 3. Write this equation in standard form:  $y = -2x^2 + 16x 28$
- 4. Simplify by adding or subtracting like terms:  $\sqrt{32w} + 2\sqrt{2w} \sqrt{128w}, w \ge 0$
- 5. Expand and simplify this expression:  $\left(2\sqrt{5} 7\sqrt{3}\right)\left(-7\sqrt{5} 6\sqrt{3}\right)$
- 6. Rationalize the denominator:  $\frac{7}{7\sqrt{5}}$
- 7. Determine the root of each equation.
  - a)  $\sqrt{2x-6} = 4$
  - b)  $3\sqrt{2x} 3 = 2\sqrt{2x} + 1$
  - c)  $\sqrt{6x+1} = \sqrt{2x-5}$
  - d)  $12 = \sqrt{168 4x}$
- 8. Factor this polynomial:  $12x^2 + 5x 28$
- 9. Factor this polynomial expression:  $48(4x-1)^2 75(2y+3)^2$
- 10. Solve this equation:  $\sqrt{x^2 + 4} + 2 = 5x$
- **11.** Solve this equation:  $(x + 2)^2 5(x + 2) 14 = 0$
- **12.** Solve this equation:  $(3x 5)^2 = 14$
- **13.** Solve this quadratic equation: 2x(x-5) = 3(x-5) + 3
- **14.** a) Determine the value of the discriminant for this equation:  $3x^2 5x 12 = 0$ 
  - b) Use the value of the discriminant to choose a solution strategy, then solve the equation.

- 15. Use a graphing calculator to graph the quadratic function  $y = -3x^2 3x + 3$ . Determine:
  - a) the intercepts
  - b) the coordinates of the vertex
  - c) the equation of the axis of symmetry
  - d) the domain of the function
  - e) the range of the function

Write your answers to the nearest hundredth, if necessary.

- 16. Determine an equation of a quadratic function with *x*-intercepts of 2 and 6, that passes through the point L(3, 12).
- 17. Determine the *x* and *y*-intercepts, the equation of the axis of symmetry, and the coordinates of the vertex of the graph of  $y = -2x^2 + 8x 6$ .
- **18.** A book store sells dictionaries for \$22. At this price, the store sells approximately 100 dictionaries per week. The store manager estimates that for every \$0.50 decrease in price, the store will sell 25 more dictionaries. Determine the price of a dictionary that will maximize the revenue.
- **19.** Represent the solution of this quadratic inequality on a number line:  $-2x^2 13x > -24$
- **20.** Graph the inequality:  $y \ge \frac{1}{4}x 3$



**21.** Graph the inequality:  $y < 2x^2 + 2$ 



- 22. Solve this quadratic-quadratic system algebraically.  $y = (x-2)^2$  $y = -2x^2 + 2x + 4$
- **23.** Given the following information about  $\triangle ABC$ , determine how many triangles can be constructed.  $a = 5.6 \text{ cm}, c = 7.8 \text{ cm}, \angle A = 38^{\circ}$
- 24. Solve  $\Delta$ UVW. Give angle measures to the nearest degree and side lengths to the nearest tenth of a centimetre.



- 25. Simplify this rational expression. State the non-permissible values of the variable.  $\frac{25-9p^2}{36p^2-100}$
- **26.** Simplify this expression:

$$\frac{7}{q-2} \cdot \frac{2q-4}{q+4}$$

**27.** Simplify.  $r^2 - r - 6$ 

$$\frac{x^2 - x - 6}{4x - 12} + \frac{x^2 + 7x + 10}{x^2 + 5x}$$

**28.** Write the absolute value function  $y = |x^2 - 3x - 10|$  in piecewise notation.



**29.** Solve this equation:  $|x^2 - 5x - 30| = 6$ 

# Problem

- **1.** Solve  $x^2 13x 7 = 0$  by completing the square. Show your work.
- 2. Sketch a graph of this quadratic function:  $y = 2x^2 x 15$ Explain your steps.



- **3.** Point P(-1, -5) is a terminal point of an angle  $\theta$  in standard position.
  - a) Determine the ratios  $\cos \theta$ ,  $\sin \theta$ , and  $\tan \theta$ .
  - b) Determine the measure of  $\theta$  to the nearest degree.

Show your work.

4. Solve  $\triangle ABC$ . Give angle measures to the nearest degree.



5. Use the graph of y = f(x) to sketch a graph of  $y = \frac{1}{f(x)}$ . Write the equation of the linear and reciprocal



- 6. The graph of the reciprocal of a quadratic function has one vertical asymptote, x = 1. Points (2, 1) and (0, 1) are common to the graphs of the quadratic function and its reciprocal.
  - a) Sketch the graphs of the quadratic function and its reciprocal on the same grid.
  - b) Determine the equations of both the quadratic function and its reciprocal.

Describe your strategy.



#### **Pre-Calc 11 Final Review**

## **Answer Section**

#### SHORT ANSWER

**1.** ANS: h = -1PTS: 0 REF: 7.5 Solving Rational Equations DIF: Moderate LOC: 11.AN6 TOP: Algebra and Number KEY: Procedural Knowledge 2. ANS: 60 km/h PTS: 0 DIF: Moderate **REF: 7.6 Applications of Rational Equations** LOC: 11.AN6 TOP: Algebra and Number KEY: Procedural Knowledge | Problem-Solving Skills **3.** ANS:  $y = -2(x-4)^2 + 4$ PTS: 0 DIF: Moderate REF: 4.5 Equivalent Forms of the Equation of a Quadratic Function LOC: 11.RF4 TOP: Relations and Functions KEY: Procedural Knowledge **4.** ANS:  $-2\sqrt{2w}$ PTS: 0 DIF: Easy REF: 2.3 Adding and Subtracting Radical Expressions LOC: 11.AN2 **TOP:** Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge **5.** ANS:  $56 + 37\sqrt{15}$ PTS: 0 DIF: Moderate REF: 2.4 Multiplying and Dividing Radical Expressions LOC: 11.AN2 **TOP:** Relations and Functions KEY: Procedural Knowledge **6.** ANS:  $7\sqrt{5}$ 35 PTS: 0 DIF: Easy REF: 2.4 Multiplying and Dividing Radical Expressions LOC: 11.AN2 **TOP:** Relations and Functions KEY: Procedural Knowledge **7.** ANS: a) x = 11b) x = 8c) The equation has no real root. d) *x* = 6 PTS: 0 DIF: Moderate **REF: 2.5 Solving Radical Equations** LOC: 11.AN3 TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge

8. ANS: (4x + 7)(3x - 4)PTS: 0 **REF: 3.1 Factoring Polynomial Expressions** DIF: Easy LOC: 11.RF1 **TOP:** Relations and Functions KEY: Procedural Knowledge **9.** ANS: 3(16x + 10y + 11)(16x - 10y - 19)PTS: 0 REF: 3.1 Factoring Polynomial Expressions DIF: Moderate LOC: 11.RF1 **TOP:** Relations and Functions KEY: Procedural Knowledge **10.** ANS:  $x = \frac{5}{6}$ PTS: 0 DIF: Moderate REF: 3.2 Solving Quadratic Equations by Factoring LOC: 11.AN3 TOP: Algebra and Number **KEY:** Procedural Knowledge **11.** ANS: x = 5 or x = -4PTS: 0 REF: 3.2 Solving Quadratic Equations by Factoring DIF: Moderate LOC: 11.RF5 **TOP:** Relations and Functions KEY: Procedural Knowledge 12. ANS:  $x = \frac{5}{3} \pm \frac{\sqrt{14}}{3}$ PTS: 0 REF: 3.3 Using Square Roots to Solve Quadratic Equations DIF: Easy LOC: 11.RF5 **TOP:** Relations and Functions KEY: Procedural Knowledge **13.** ANS:  $x = \frac{13 \pm \sqrt{73}}{4}$ **PTS:** 0 DIF: Moderate REF: 3.4 Developing and Applying the Quadratic Formula LOC: 11.RF5 **TOP:** Relations and Functions KEY: Procedural Knowledge **14.** ANS: a)  $b^2 - 4ac = 169$ b) The discriminant is a perfect square, so use factoring.  $x = -\frac{4}{3}$  or x = 3PTS: 0 DIF: Moderate REF: 3.5 Interpreting the Discriminant LOC: 11.RF5 **TOP:** Relations and Functions

ID: A

KEY: Conceptual Understanding | Procedural Knowledge

- a) *x*-intercepts: -1.62 and 0.62 *y*-intercept: 3
- b) vertex: (-0.5, 3.75)
- c) axis of symmetry: x = -0.5
- d) domain:  $x \in \mathbb{R}$
- e) range:  $y \le 3.75, y \in \mathbb{R}$

PTS:0DIF:ModerateREF:4.1 Properties of a Quadratic FunctionLOC:11.RF4TOP:Relations and Functions

- KEY: Conceptual Understanding | Procedural Knowledge
- **16.** ANS:

 $y = -4(x-4)^2 + 16$ 

PTS:0DIF:DifficultREF:4.4 Analyzing Quadratic Functions of the Form  $y = a(x - p)^2 + q$ LOC:11.RF3TOP:Relations and FunctionsKEY:Conceptual Understanding | Procedural Knowledge

## **17.** ANS:

*y*-intercept: -6*x*-intercepts: 1 and 3 equation of the axis of symmetry: x = 2coordinates of the vertex: (2, 2)

PTS: 0 DIF: Moderate

REF: 4.6 Analyzing Quadratic Functions of the Form  $y = ax^2 + bx + c$ 

- LOC: 11.RF4 TOP: Relations and Functions
- KEY: Conceptual Understanding | Procedural Knowledge

## **18.** ANS:

\$12

PTS: 0 DIF: Moderate

REF: 4.7 Modelling and Solving Problems with Quadratic Functions

- LOC: 11.RF4 TOP: Relations and Functions
- KEY: Problem-Solving Skills | Procedural Knowledge
- **19.** ANS:

The solution is:  $-8 < x < 1.5, x \in \mathbb{R}$ 

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	-9	-8	-7	-6	-5	4	-3	-2	-1	0	1	2	3	4	5	

PTS:	0	DIF:	Moderate	REF:	5.1 Solving	Quadratic	Inequalities i	n One Variable
LOC:	11.RF8	TOP:	Relations and I	Functio	ons	KEY:	Procedural K	nowledge

**20.** ANS:



PTS:0DIF:ModerateREF:5.2 Graphing Linear Inequalities in Two VariablesLOC:11.RF7TOP:Relations and FunctionsKEY:Conceptual Understanding | Procedural Knowledge

**21.** ANS:



PTS:0DIF:ModerateREF:5.3 Graphing Quadratic Inequalities in Two VariablesLOC:11.RF7TOP:Relations and FunctionsKEY:Conceptual Understanding | Procedural Knowledge

- NET: Conceptual Understanding | Procedural
- **22.** ANS:

The solutions are: (2, 0) and (0, 4)

PTS:0DIF:ModerateREF:5.5 Solving Systems of Equations AlgebraicallyLOC:11.RF6TOP:Relations and FunctionsKEY:Procedural Knowledge

Two triangles can be constructed.

PTS: 0 REF: 6.4 The Sine Law DIF: Easy **TOP:** Trigonometry LOC: 11.T3 KEY: Conceptual Understanding | Procedural Knowledge 24. ANS:  $\angle W = 34^{\circ}$  $\angle U = 66^{\circ}$ VW = 6.5 cmPTS: 0 DIF: Moderate REF: 6.4 The Sine Law TOP: Trigonometry LOC: 11.T3 KEY: Conceptual Understanding | Procedural Knowledge 25. ANS:  $\frac{25 - 9p^2}{36p^2 - 100} = -\frac{1}{4}$ The non-permissible values are  $p = \frac{5}{3}$  and  $p = -\frac{5}{3}$ . PTS: 0 DIF: Moderate REF: 7.1 Equivalent Rational Expressions LOC: 11.AN4 TOP: Algebra and Number KEY: Conceptual Understanding | Procedural Knowledge 26. ANS:  $\frac{14}{q+4}, q \neq 2, q \neq -4$ PTS: 1 DIF: Easy **REF: 7.2 Multiplying and Dividing Rational Expressions** LOC: 11.AN5 TOP: Algebra and Number KEY: Conceptual Understanding | Procedural Knowledge 27. ANS:  $\frac{(x+2)(x+4)}{4x}, x \neq 0, x \neq 3, x \neq -5$ PTS: 0 DIF: Moderate REF: 7.4 Adding and Subtracting Rational Expressions with Binomial and Trinomial Denominators LOC: 11.AN5 TOP: Algebra and Number KEY: Conceptual Understanding | Procedural Knowledge 28. ANS:  $y = \begin{cases} x^2 - 3x - 10, & \text{if } x \le -2 \text{ or } x \ge 5 \\ -(x^2 - 3x - 10), & \text{if } -2 < x < 5 \end{cases}$ PTS: 0 DIF: Moderate **REF: 8.1 Absolute Value Functions** LOC: 11.RF2 **TOP:** Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge

The solutions are: x = 9, x = -4, x = -3, and x = 8

PTS:0DIF:ModerateREF:8.2 Solving Absolute Value EquationsLOC:11.RF2TOP:Relations and FunctionsKEY:Conceptual Understanding | Procedural Knowledge

## PROBLEM

**1.** ANS:

$$x^{2} - 13x - 7 = 0$$
  

$$x^{2} - 13x = 7$$
  

$$x^{2} - 13x + \frac{169}{4} = 7 + \frac{169}{4}$$
  

$$(x - \frac{13}{2})^{2} = \frac{197}{4}$$
  

$$x - \frac{13}{2} = \pm \sqrt{\frac{197}{4}}$$
  

$$x = \frac{13}{2} \pm \sqrt{\frac{197}{4}}$$
  

$$x = \frac{13 \pm \sqrt{197}}{2}$$
  
The roots are:  $x = \frac{13 + \sqrt{197}}{2}$  and  $x = \frac{13 - \sqrt{197}}{2}$ 

PTS:0DIF:ModerateREF:3.3 Using Square Roots to Solve Quadratic EquationsLOC:11.RF5TOP:Relations and FunctionsKEY:Communication | Problem-Solving Skills

Check whether the equation factors. The value of the discriminant is:  $(-1)^2 - 4(2)(-15) = 121$ Since 121 is a perfect square, the equation factors. Use decomposition to factor.  $y = 2x^2 - x - 15$  y = (2x + 5)(x - 3)The *x*-intercepts are:  $-\frac{5}{2}$  and 3, or -2.5 and 3 The *x*-coordinate of the vertex is:  $\frac{-2.5 + 3}{2} = 0.25$ Determine the *y*-coordinate of the vertex. Substitute x = 0.25 in  $y = 2x^2 - x - 15$ .  $y = 2x^2 - x - 15$   $y = 2(0.25)^2 - (0.25) - 15$  y = -15.125The coordinates of the vertex are: (0.25, -15.125)

On a grid, mark points at the vertex and the intercepts. Draw a smooth curve through the points.



PTS:0DIF:ModerateREF:4.6 Analyzing Quadratic Functions of the Form  $y = ax^2 + bx + c$ LOC:11.RF4TOP:Relations and FunctionsKEY:Communication | Procedural Knowledge

- a) Determine the distance r from the origin to P. x = -1, y = -5  $r = \sqrt{x^2 + y^2}$   $r = \sqrt{(-1)^2 + (-5)^2}$   $r = \sqrt{26}$   $\cos \theta = \frac{x}{r}$   $\cos \theta = \frac{-1}{\sqrt{26}}$   $\sin \theta = \frac{y}{r}$   $\sin \theta = \frac{-5}{\sqrt{26}}$   $\tan \theta = \frac{y}{x}$   $\tan \theta = \frac{-5}{-1}, \text{ or } 5$
- b) The reference angle is:  $\tan^{-1}(5) = 78.69...^{\circ}$ Since  $\theta$  is in Quadrant 3, the angle  $\theta$  is approximately:  $180^{\circ} + 78.69^{\circ} = 258.69^{\circ}$

PTS: 0 DIF: Moderate REF: 6.2 Angles in Standard Position in All Quadrants LOC: 11.T2 TOP: Trigonometry KEY: Procedural Knowledge | Communication

Use:  $b^2 = a^2 + c^2 - 2ac \cos B$ Substitute: a = 9, b = 10, c = 5 $10^2 = 9^2 + 5^2 - 2(9)(5) \cos B$  $\cos B = \frac{9^2 + 5^2 - 10^2}{2(9)(5)}$  $\angle B = \cos^{-1}\left(\frac{9^2 + 5^2 - 10^2}{2(9)(5)}\right)$ ∠B = 86.1774...  $\angle B \doteq 86^{\circ}$ Use:  $c^2 = a^2 + b^2 - 2ab \cos C$ Substitute: a = 9, b = 10, c = 5 $5^2 = 9^2 + 10^2 - 2(9)(10) \cos C$  $\cos C = \frac{9^2 + 10^2 - 5^2}{2(9)(10)}$  $\angle C = \cos^{-1}\left(\frac{9^2 + 10^2 - 5^2}{2(9)(10)}\right)$ ∠C = 29.9264...  $\angle C \doteq 30^{\circ}$  $\angle A \doteq 180^{\circ} - (86.1774...^{\circ} + 29.9264...^{\circ})$  $\angle A \doteq 64^{\circ}$ 

PTS: 1 DIF: Moderate REF: 6.5 The Cosine Law LOC: 11.T3 TOP: Trigonometry KEY: Conceptual Understanding | Procedural Knowledge

An equation of the line has the form y = mx + b. Use the points (0, 0) and (2, 8) to determine *m* and *b*.

$$m = \frac{8-0}{2-0}$$
$$= 4$$
$$y = 4x + b$$
$$8 = 4(2) + b$$

b = 0

So, an equation of the linear function is y = 4x. For the graph of the reciprocal function:

The equation is:  $y = \frac{1}{4x}$ 

Horizontal asymptote: y = 0*x*-intercept is 0, so vertical asymptote is x = 0.



Mark points at y = 1 and y = -1 on the graph of y = 4x. Draw a smooth curve through each point so that the curve approaches the asymptotes but never touches them.

PTS:0DIF:DifficultREF:8.3 Graphing Reciprocals of Linear FunctionsLOC:11.RF11TOP:Relations and FunctionsKEY:Conceptual Understanding | Procedural Knowledge | Communication

- **6.** ANS:
  - a) When the graph of a reciprocal function has one vertical asymptote, the graph of the corresponding quadratic function has one *x*-intercept. Since the vertical asymptote is x = 1, the graph of the quadratic function has vertex (1, 0) and passes through the points (2, 1) and (0, 1). Since the points common to both graphs are above the *x*-axis, the graph of the quadratic function opens up.



b) The equation of the quadratic function has the form y = a(x - h)<sup>2</sup> + k, where (h, k) is the vertex of the parabola, and *a* represents its size and direction. Substitute h = 1 and k = 0.
y = a(x - (1))<sup>2</sup> + 0
y = a(x - 1)<sup>2</sup>
Use one of the points (2, 1) and (0, 1) to solve for a: a = 1

The equation of the reciprocal function is:  $y = (x - 1)^2$ The equation of the reciprocal function is:  $y = \frac{1}{(x - 1)^2}$ 

PTS:0DIF:DifficultREF:8.5 Graphing Reciprocals of Quadratic FunctionsLOC:11.RF11TOP:Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge | Communication