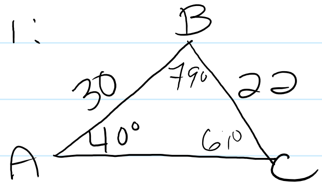
Sine Law - Ambiguous Case

Tuesday, November 19, 2019 9:05 AM

Solve $\triangle ABC$ with $\angle A = 40°$ side $c = 30$cm and side $a = 22$ cm

Case 1:



$$\frac{\sin 40}{22} = \frac{\sin C}{30}$$

$$\sin C = \frac{30\sin 40}{22} = .8765$$

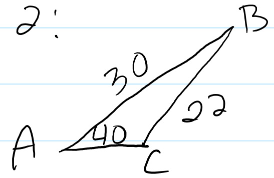
$$\angle C = \sin^{-1}(.8765) = 61°$$

$$\angle B = 180 - 40 - 61 = 79°$$

$$\frac{\sin 40}{22} = \frac{\sin 79}{b}$$

$$b = \frac{22\sin 79}{\sin 40} = 33.6 \text{ cm}$$

Case 2:

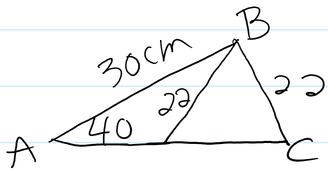


$$\angle C = 180 - 61 = 119°$$

$$\angle B = 180 - 40 - 119 = 21°$$

$$\frac{\sin 40}{22} = \frac{\sin 21}{b}$$

$$b = \frac{22\sin 21}{\sin 40} = 12.3 \text{ cm.}$$

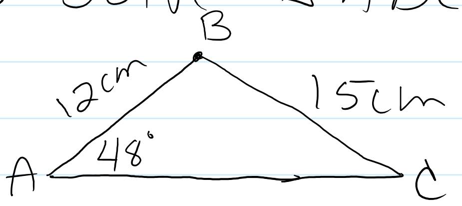


To have an ambiguous case you must have angle - side - side.
- the side opposite the given must be less than the other given side but longer than the height to have 2 triangles.

For the following state, how many

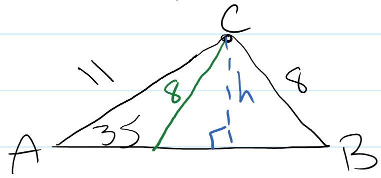
triangles are possible + why.

① Solve $\triangle ABC$ for $\angle A = 48°$, $c = 12cm$ $a = 15m$.



1 Δ possible because side opposite given $\angle$ is > other given side.

② Solve $\triangle ABC$ for $\angle A = 35°$, side $a = 8$ cm, side $b = 11$ cm.



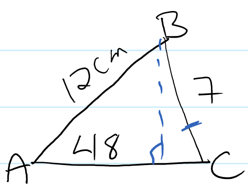
$\sin 35 = \frac{h}{11}$

$h = 11 \sin 35$

$h = 6.3.$

2 Δ's are possible because side opposite given $\angle$ is > height and < other given side

③ Solve $\triangle ABC$, for $\angle A = 48°$, $c = 12cm$, $a = 7cm$.

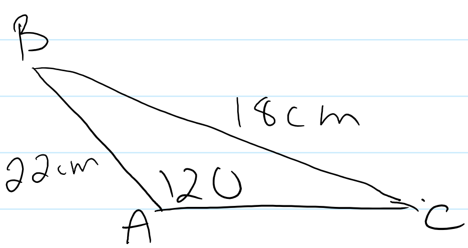


$\sin 48 = \frac{h}{12}$

$h = 12 \sin 48$

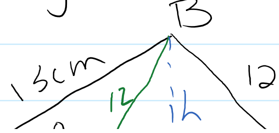
$h = 8.9cm$

No Δ's are possible because the side opposite the given $\angle$ is < height.

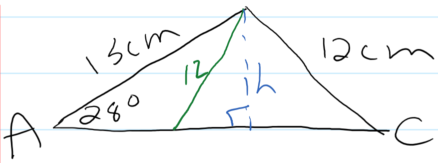


no Δ's because side opposite an obtuse $\angle$ must be longest side in Δ.

Try: Solve $\triangle ABC$ for $\angle A = 28°$, $C = 15cm$, $a = 12cm$



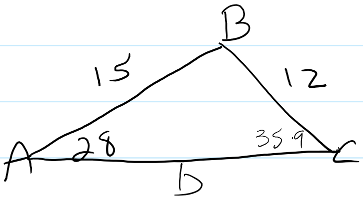
$\sin 28 = \frac{h}{15}$



$\sin 28 = \frac{h}{15}$

$h = 15\sin 28 = 7 \quad \therefore$ 2 Δ's possible.

Case 1 :



$$\frac{\sin 28}{12} = \frac{\sin C}{15}$$

$$\sin C = \frac{15\sin 28}{12} = .5868$$

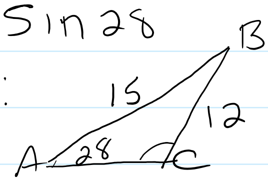
$$\angle C = \sin^{-1}(.5868) = 35.9°$$

$$\angle B = 180 - 28 - 35.9 = 116.1°$$

$$\frac{\sin 28}{12} = \frac{\sin 116.1}{b}$$

$$b = \frac{12\sin 116.1}{\sin 28} = 23.0 \text{ cm.}$$

Case 2 :



$$\angle C = 180 - 35.9 = 144.1°$$

$$\angle B = 180 - 144.1 - 28 = 7.9°$$

$$\frac{\sin 28}{12} = \frac{\sin 7.9}{b}$$

$$b = \frac{12\sin 7.9}{\sin 28} = 3.5 \text{ cm}$$

Handout + Pg 477-486 # 6, 7, 9b, 15, 17