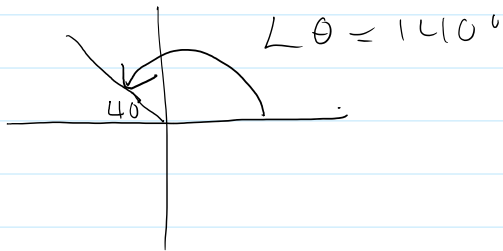
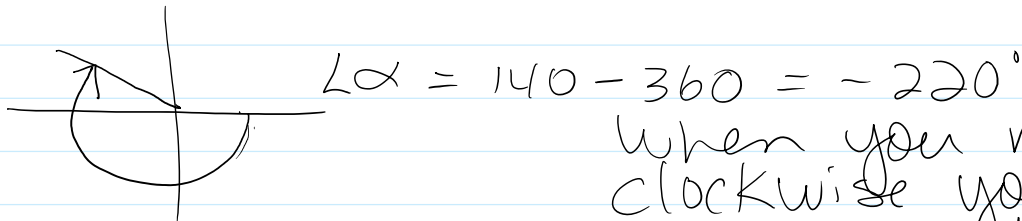


Co-terminal Angles

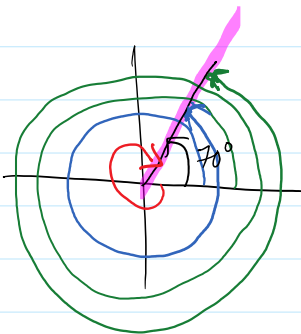
Friday, November 15, 2019 11:31 AM



When you move counter-clockwise your angles are positive



When you move clockwise your angles are negative



$$\begin{aligned}\angle \theta &= 70^\circ \\ &= 70 - 360 = -290^\circ \\ &= 70 + 360 = 430^\circ \\ &= 70 + 360 + 360 = 790^\circ\end{aligned}$$

We call these co-terminal angles because they all share the same terminal arm.

In general the co-terminals above could be expressed as $70 + k360$ where $k \in \mathbb{I}$.

Eg Find all angles between 0° and 2000° that are co-terminal to $\angle \theta = 190^\circ$ in standard position

- ① $190 + 360 = 550^\circ$
- ② $550 + 360 = 910^\circ$
- ③ $910 + 360 = 1270^\circ$

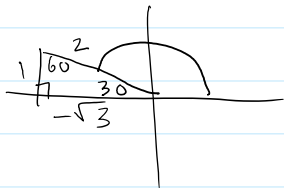
$$\begin{aligned} \textcircled{4} \quad 1270 + 360 &= 1630^\circ \\ \textcircled{5} \quad 1630 + 360 &= 1990^\circ \\ &\text{in general } 190 + k360 \end{aligned}$$

The angle in standard position is called the principal angle. The principal angle is the smallest positive angle that is co-terminal to the given angle. It will always be between 0° and 360° .

Find the principal angle for a) -840° b) 1270°

$$\begin{aligned} \text{a) } -840 + (3)360 &= 240^\circ \\ \text{b) } 1270 - (3)360 &= 190^\circ \end{aligned} \left. \vphantom{\begin{aligned} \text{a) } -840 + (3)360 \\ \text{b) } 1270 - (3)360 \end{aligned}} \right\} \text{principal angles}$$

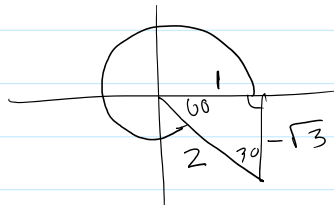
Eg Find the exact trig ratios for $\angle \alpha = -930^\circ$
 ① Find the principal angle $-930 + 3(360) = 150$



$$\cos -930 = -\frac{\sqrt{3}}{2} \quad \sin -930 = \frac{1}{2}$$

$$\tan -930 = \frac{1}{-\sqrt{3}}$$

Try: Find the exact trig ratios for $\angle \alpha = 1020^\circ$
 Principal $\angle = 1020 - 2(360) = 300^\circ$

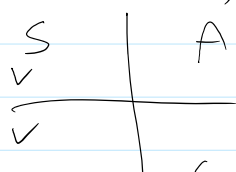


$$\cos 1020 = \frac{1}{2} \quad \sin 1020 = -\frac{\sqrt{3}}{2}$$

$$\tan 1020 = \frac{-\sqrt{3}}{1} \text{ or } -\sqrt{3}$$

Eg $\cos \alpha = -\frac{3}{5}$

a) Find the other primary trig ratios



$$\begin{aligned} y &= \sqrt{r^2 - x^2} \\ y &= \sqrt{5^2 - (-3)^2} \\ y &= \sqrt{16} \end{aligned}$$

QII $x = -3, y = 4, r = 5$

$$\sin \alpha = \frac{4}{5} \quad \tan \alpha = \frac{4}{-3}$$

QIII $x = -3, y = -4, r = 5$

v		$y = 0 - (-3)$
v		$y = \sqrt{16}$
T	C	$y = 4$

$(\sin \alpha = \frac{4}{5}) \quad \tan \alpha = \frac{4}{-3}$
 QIII $x = -3, y = -4, r = 5$
 $\sin \alpha = -\frac{4}{5} \quad \tan \alpha = \frac{4}{-3}$

b) Find all co-terminal angles when $-360^\circ \leq \alpha \leq 720^\circ$

$\angle \theta_{\text{ref}} = \sin^{-1}(\frac{4}{5}) = 53^\circ$

QII $\angle \theta = 180 - 53 = 127^\circ$

QIII $\angle \theta = 180 + 53 = 233^\circ$

Negative: $127 - 360 = -233$

$233 - 360 = -127$

Positive: $127 + 360 = 487$

$233 + 360 = 593$

$\angle \alpha = -233^\circ, -127^\circ, 127^\circ, 233^\circ, 487^\circ, 593^\circ$

Pg 445-454 # 4-20