Applications of Rational Equations

Wednesday, December 4, 2019 1:45 PM

The average speed of a car is 3 times the average speed of a cyclist. It would take a cyclist 20 hrs longer to drive 900 km than the car. Find the average speed of both.

$s \cdot t = d$ $\quad t = \frac{d}{s}$

	Speed	Time	Distance
car	$3x$	$\frac{900}{3x}$	900
cyclist	x	$\frac{900}{x}$	900

Car Cyclist

$$\frac{900}{3x} + 20 = \frac{900}{x}$$

$$900 + 60x = 2700$$
$$-900 \qquad -900$$
$$\frac{60x}{60} = \frac{1800}{60}$$
$$x = 30$$

The cyclist's speed is 30 km/h + the car's speed is 90km/h

Eg A boat travels 70 km up a river with a current of 2km/h. It then travels back down to its starting point. If the whole trip takes 12 hours, what is the boat's speed in still water?

Let x = boat's speed in still water

	Speed	Time $= \frac{d}{s}$	Distance
upstream	$x-2$	$\frac{70}{x-2}$	70
downstream	$x+2$	$\frac{70}{x+2}$	70

$$\frac{70}{x-2} + \frac{70}{x+2} = 12 \quad (x-2)(x+2)$$

$$70(x+2) + 70(x-2) = 12(x^2-4)$$
$$70x + 140 + 70x - 140 = 12x^2 - 48$$
$$140x = 12x^2 - 48$$

$0 = 12x^2 - 140x - 48$

$0 = 4(3x^2 - 35x - 12)$

$0 = 4(3x + 1)(x - 12)$

Speed can't be negative $x = -\frac{1}{3}, 12$

The boat's speed in still water is 12 km/h.

Try: A speed boat can travel 108 km downstream in the same time it can travel 78 km upstream. If the speed of the boat in still water is 62 km/h what is the speed of the current?

Let x = speed of current

	Speed	Time = D/S	Distance
Downstream	$62+x$	$\frac{108}{62+x}$	108
Upstream	$62-x$	$\frac{78}{62-x}$	78

$$\frac{108}{62+x} = \frac{78}{62-x}$$

$$108(62-x) = 78(62+x)$$

$$6696 - 108x = 4836 + 78x$$

-4836 $+108x$

$$\frac{1860}{186} = \frac{186x}{186}$$

$$10 = x$$

Speed of current is 10 km/h.

Work Problems.

Eg It takes Jerry 9 hrs longer to paint a house than George. Together they can paint the whole house in 20 hours. How long would it take each of

them to paint it by themselves?

Let x = hrs for George alone
$x+9$ = hrs for Jerry alone

George paints $\frac{1}{x}$ of house per hour.
In 20 hours George paints $20 \times \frac{1}{x} = \frac{20}{x}$ of the house

Jerry paints $20 \times \frac{1}{x+9} = \frac{20}{x+9}$ of the house in 20 hrs.

$$\frac{20}{x}^{x(x+9)} + \frac{20}{x+9}^{x(x+9)} = 1^{x(x+9)}$$

$$20(x+9) + 20x = x(x+9)$$
$$20x + 180 + 20x = x^2 + 9x$$
$$180 + 40x = x^2 + 9x$$
$$0 = x^2 - 31x - 180$$
$$0 = (x-36)(x+5)$$
$$x = 36, \cancel{-5}$$

It takes George 36 hrs alone + Jerry 45 hrs alone.

Try: Jane works twice as fast as her daughter Anna. If it takes them 15 min to clean the kitchen, how long would it take each of them alone?

Let x = Jane's minutes alone $\frac{1}{x} \cdot 15 = \frac{15}{x}$
$2x$ = Anna's minutes alone $\frac{1}{2x} \cdot 15 = \frac{15}{2x}$

$$\frac{15}{x}^{2x} + \frac{15}{2x}^{2x} = 1^{2x}$$

$$30 + 15 = 2x$$
$$\frac{45}{2} = \frac{2x}{2}$$
$$22.5 = x$$

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It takes Jane 22.5 min alone & Anna 45 min. alone.