Powers with Negative Rational Exponents

Thursday, January 30, 2020 10:20 AM

Review:

$$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$$

$$\left(\frac{9}{16}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{9}{16}}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\left(\frac{2}{3}\right)^{-4} \quad \left(\frac{3}{2}\right)^4 = \frac{81}{16}$$

$$4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{\left(\sqrt{4}\right)^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$4^{-\frac{3}{2}} = \left(\frac{1}{4}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{1}{4}}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$9^{-1.5} = 9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{\left(\sqrt{9}\right)^3} = \frac{1}{(3)^3} = \frac{1}{27}$$

In general $x^{-\frac{n}{m}} = \frac{1}{x^{\frac{n}{m}}} = \frac{1}{\sqrt[m]{x^n}}$ or $\frac{1}{\left(\sqrt[m]{x}\right)^n}$

Try: Write as a radical, then evaluate.

① $(-27)^{-\frac{2}{3}} = \frac{1}{(-27)^{\frac{2}{3}}} = \frac{1}{\left(\sqrt[3]{-27}\right)^2} = \frac{1}{(-3)^2} = \frac{1}{9}$ $\quad (-3)^{-2} = \frac{1}{(-3)^2}$

① $(-27)^{\,3} = \frac{1}{(-27)^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{-27})^2} = \frac{1}{(-3)^2} = \frac{1}{9}$

$(-3) = \frac{1}{(-3)^2}$

this stays negative!

② $-4^{-2.5} = -4^{-\frac{5}{2}} = \frac{-1}{4^{\frac{5}{2}}} = \frac{-1}{(\sqrt{4})^5} = \frac{-1}{2^5} = \frac{-1}{32}$

$(-27)^{-\frac{2}{3}} = (\sqrt[3]{-27})^{-2} = (-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

$\frac{1}{-4} = \frac{-1}{4} = -\frac{1}{4}$

eg $\left(\frac{9}{25}\right)^{-\frac{3}{2}} = \left(\frac{25}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{25}{9}}\right)^3 = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$

flip base only, don't flip exponent!!

$\left(\frac{-8}{27}\right)^{-\frac{4}{3}} = \left(\frac{-27}{8}\right)^{\frac{4}{3}} = \left(\sqrt[3]{\frac{-27}{8}}\right)^4 = \left(\frac{-3}{2}\right)^4 = \frac{81}{16}$

base stays negative!

In general $\left(\frac{x}{y}\right)^{-\frac{m}{n}} = \left(\frac{y}{x}\right)^{\frac{m}{n}} = \sqrt[n]{\left(\frac{y}{x}\right)^m}$ or $\left(\sqrt[n]{\frac{y}{x}}\right)^m$

Try ① $\left(\frac{-27}{64}\right)^{-\frac{2}{3}} = \left(\frac{-64}{27}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{-64}{27}}\right)^2 = \left(\frac{-4}{3}\right)^2 = \frac{16}{9}$

② $\left(\frac{49}{100}\right)^{-\frac{3}{2}} = \left(\frac{100}{49}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{100}{49}}\right)^3 = \left(\frac{10}{7}\right)^3 = \frac{1000}{343}$

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